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Критические переходы в климатической системе. Сигналы раннего предупреждения

Climate tipping points. Early warning signals

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Вторые научные чтения памяти Георгия Вадимовича Грузы «СТАТИСТИЧЕСКАЯ
КЛИМАТОЛОГИЯ И МОНИТОРИНГ КЛИМАТА»

12 марта 2026 г.



Climate tipping point – критический порог воздействия, за которым климатическая система реорганизуется, обычно резко и необратимо. Abrupt Climate Changes. IPCC A6 WGI AnnexVII

Tipping Elements -

1. Atlantic Meridional Overturning Circulation (АМОС)
2. Atlantic Subpolar Gyre
3. Ледниковый щит Гренландии (и Западной Антарктиды)
4. Вечная мерзлота
5. Зимний дрейфующий лёд СЛО
6. Бореальные леса
7. Леса Амазонии
8. Муссонная циркуляция

- I. Происходят относительно быстро
- II. Определяются **нелинейными процессами**
- I. Необратимы (обычно)

Временные масштабы ТР -

Дрейфующие льды Арктики – годы (?),
Atlantic Meridional Overturning Circulation – десятилетия или столетия,
Ледниковые щиты – столетия и тысячелетия



Atlantic Meridional Overturning Circulation and North Atlantic Deep Water Formation

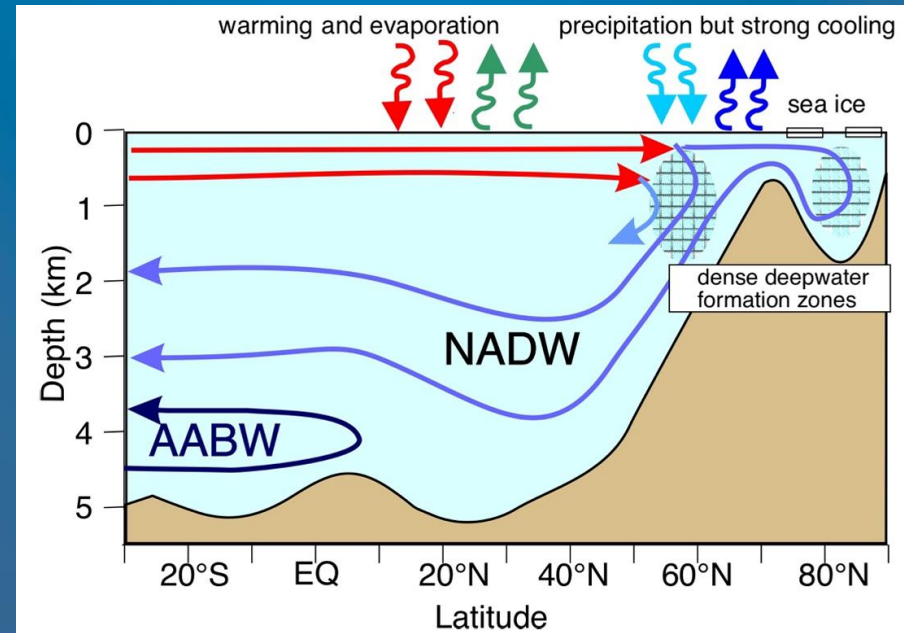
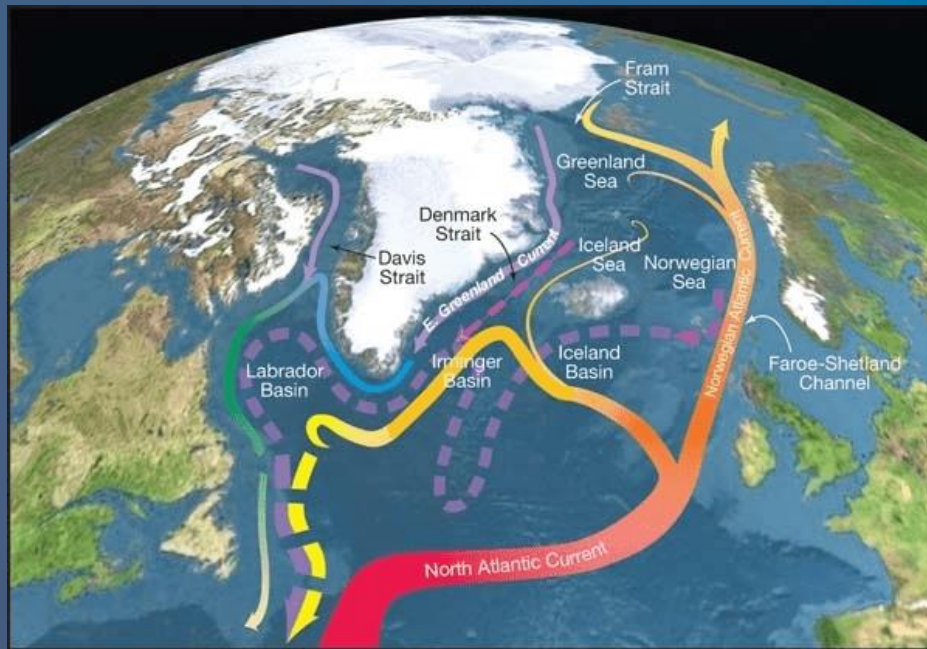


Figure 5. Schematic representation of AMOC and NADW formation.

Surface currents (solid curves) and deep currents (dashed curves) that form a portion of the Atlantic meridional overturning circulation.

Seidov et al,
2025, *Oceans*



Палеоклиматические аналоги – события Dansgaard-Oeschger. Last Glacial Period 115-12 Ky bp

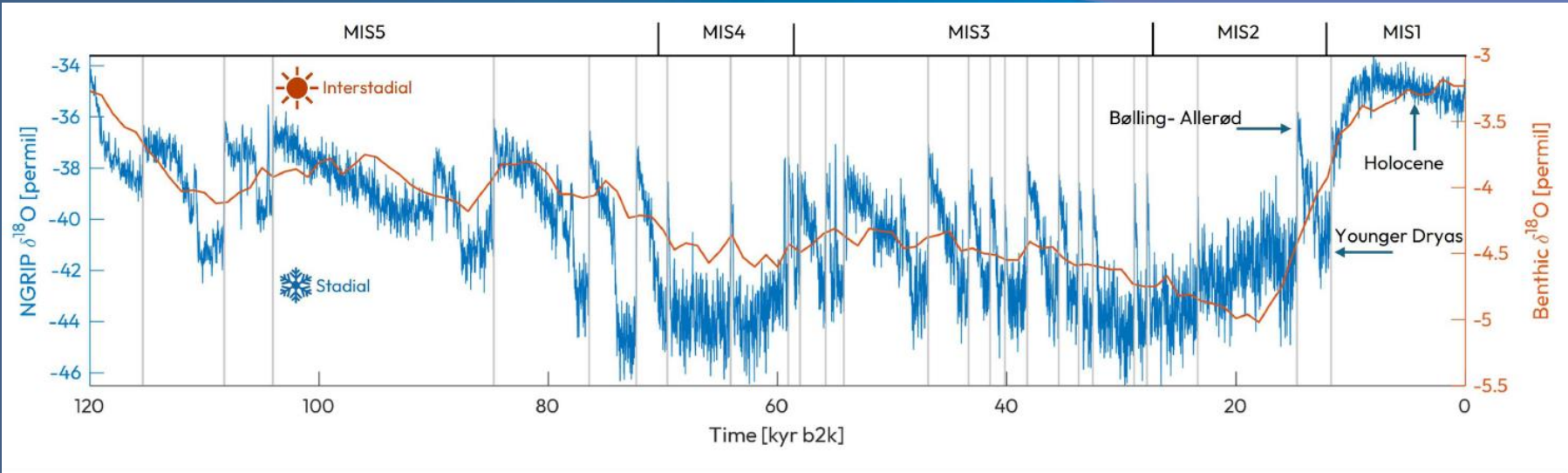


FIG. 1. Variability of the stable oxygen isotope ratio ($\delta^{18}\text{O}$) during the last glacial period. In this figure and throughout the paper, kyr b2k refers to “millennia before the year 2000.” The blue line indicates the Northern Greenland Ice Core Project (NGRIP) $\delta^{18}\text{O}$ data with regular sampling of 20 years.⁹ The ratio of stable oxygen isotopes ^{16}O and ^{18}O ($\delta^{18}\text{O}$) is often used as a proxy for atmospheric temperature at the site of the ice core, with more negative values indicating colder temperatures and less negative values indicating warmer temperatures. The orange line is the benthic record¹⁰ of $\delta^{18}\text{O}$, which is a proxy for total global ice volume and, thus, a proxy for the background global mean temperature. Thin gray lines indicate the observed DO events. Marine Isotope Stages (MISs) are indicated at the top of the figure.

Переход от ледниковой (stadial) к межледниковой фазе (interstadial) во время событий D-O происходил за несколько десятилетий. Температура воздуха в регионе Северной Атлантики вырастала на 5-15 К за время жизни одного поколения (20-30 лет).

Hobden, Ashwin & Ritchie, 2025, *Chaos*

Simulated Shutdown of the AMOC



Henry Stommel, 1961 – box model

CGCM, Hosing experiments

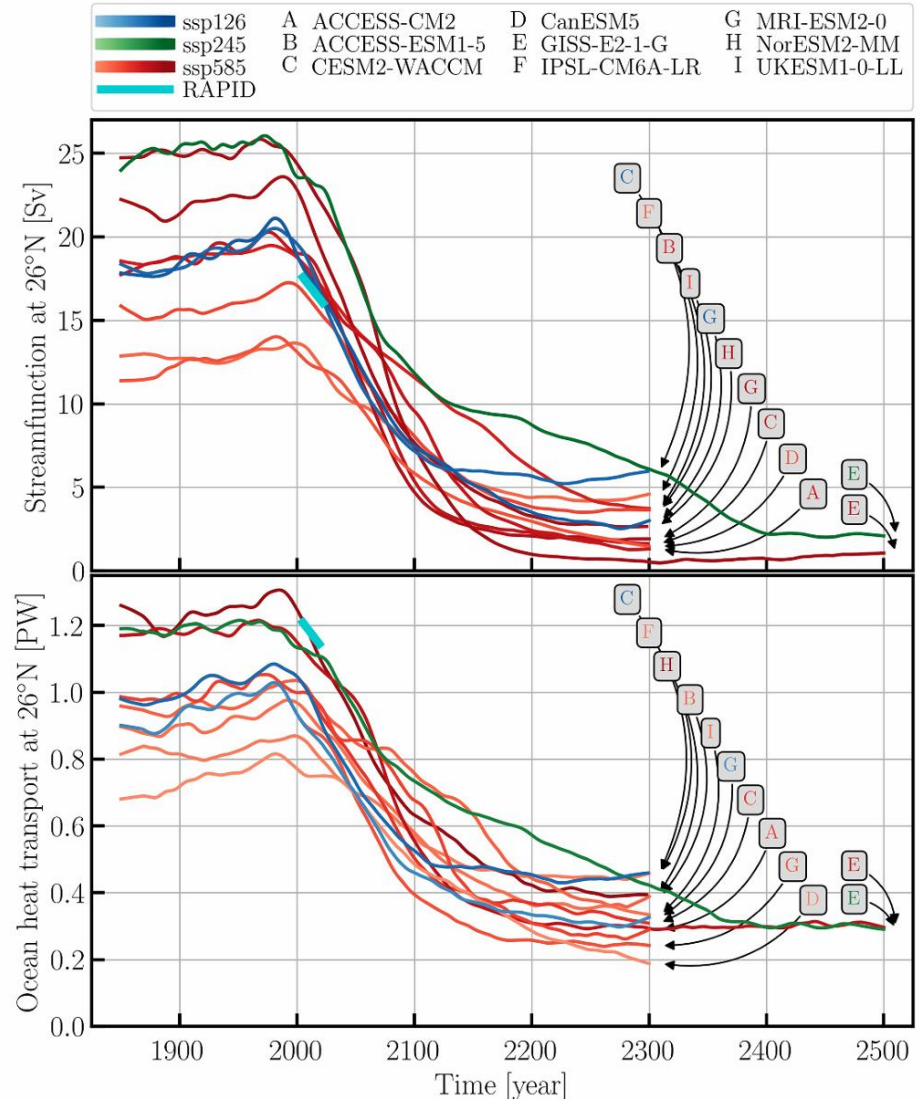
Hawkins et al, 2011, *GRL*

Jackson & Wood, 2018, *GRL*

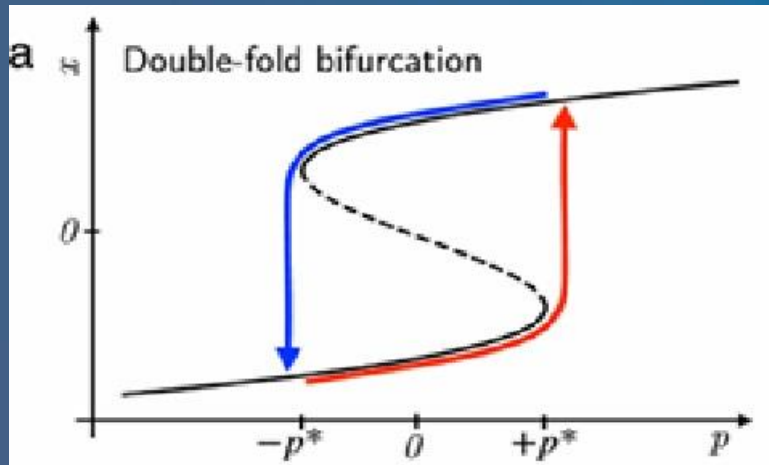
CMIP6 scenario experiments.
Shutdown after 2100.
NO prescribed freshwater perturbations

AMOC strength and heat transport, at 26°N
Drijfhout S. et al, 2025, *Environ. Res. Lett.*

AMOC strength and heat transport for NADW shutdown simulations



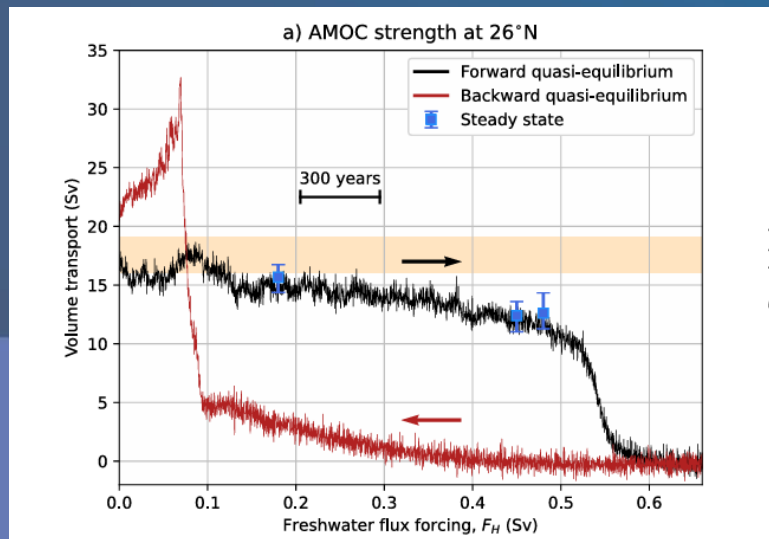
Tipping Points - Bifurcation mechanisms, 1



Bifurcation (B) Tipping points

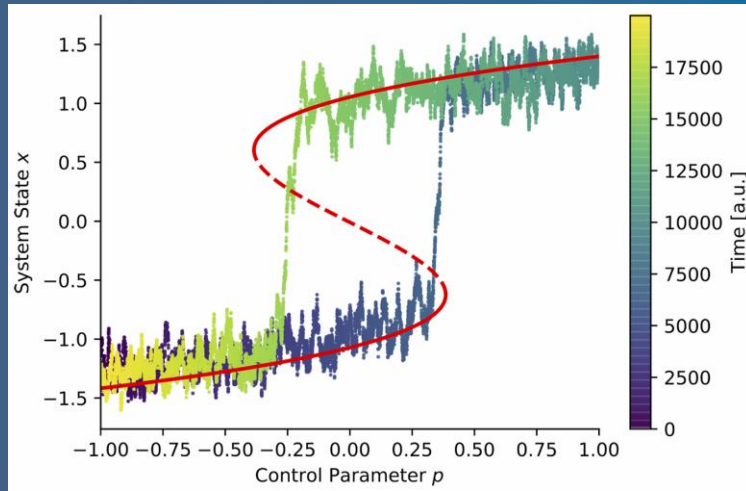
Double-fold bifurcation

Boer N., Ghil M., Stocker T, 2022,
Environ. Res. Lett.



van Westen, Vanderborght, Kliphuis & Dijkstra,
2025, *JGR: Oceans*

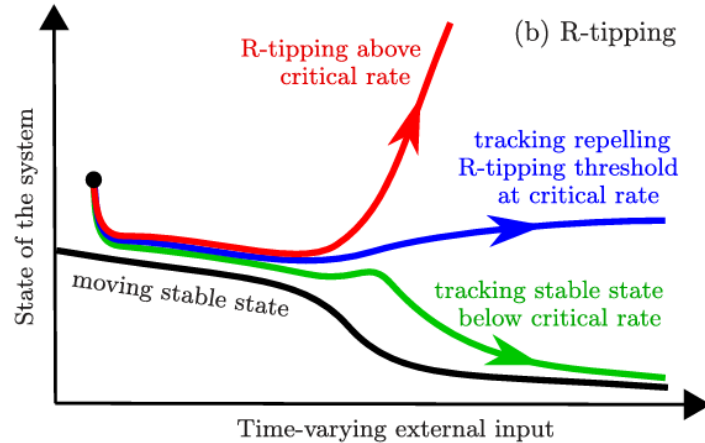
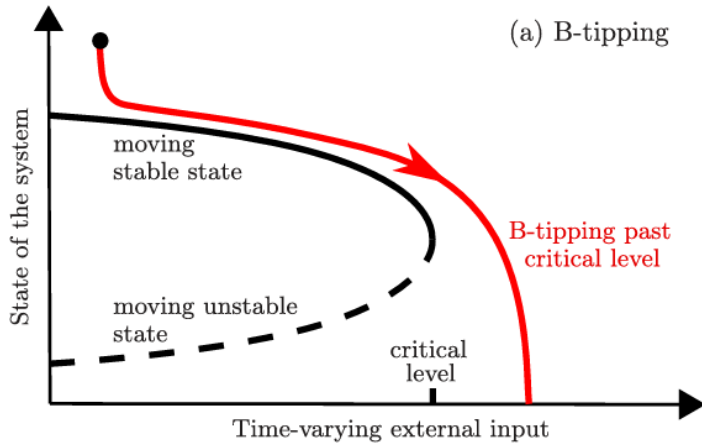
Community Earth System Model (CESM)



Noise (N) Tipping Points

Boer, Ghil, Stocker, 2022,
Environ. Res. Lett.

Rate (R) Tipping Points



Wieczorek et al, 2023,
Nonlinearity



S - Shock Tipping Point

Introduced by **Ulrike Feudel, 2021**

Klose, Donges, Feudel & Winkelmann, 2024, *Earth System Dynamics*

Shock Tipping Point – переход в новое статистически стационарное состояние в результате единичного катастрофического внешнего воздействия

Brunetti & Ragon, 2023, *Phys. Rev. E*, suggested that S-tippings could explain the events on the Cretaceous-Paleocene boundary 66.5 ml ya.

Неопротерозой, 720-635 ml ya. (Sturtian и Marinoan)

Shock Tipping Points in ESM simulations

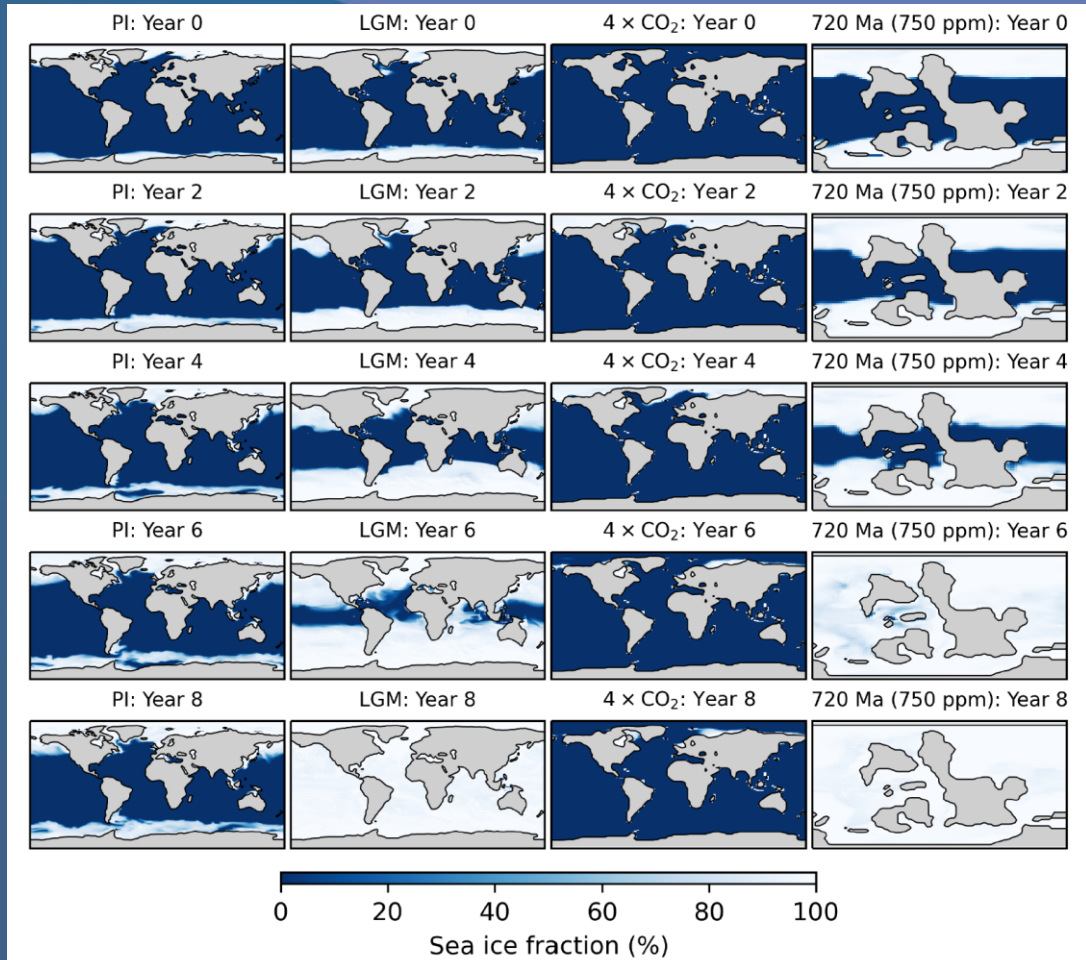


Fig. 3. Snapshots of sea ice coverage during the decade following the 200-Gt radiative forcing scenario for selected experiments. From left to right, columns show the sea ice response of the preindustrial (PI), LGM, 4 × CO₂, and 720-Ma (750-ppm) simulations, respectively.

Community Earth System Model version 1 (CESM1)

Stratospheric SO₂ injection 200 Gt.

Fu, Abbot, Koeberl & Fedorov A., 2024, *Science Advances*



Какие ещё бифуркации?

I. Бифуркации Хопфа (Subcritical & Supercritical)

II. Бифуркации коразмерности 2

1. Fold-Hopf bifurcation
2. Bogdanov-Takens bifurcation

Sinet, Ashwin, von der Heydt &
Dijkstra, 2024, *Earth System
Dynamics*

Alkhayuon et al, 2019,
Proceedings of the Royal Society A

Критические переходы (ТР) в модифицированной системе Лоренца (MLS)



$$\frac{\partial X}{\partial \tau} = \sigma Y - \sigma X,$$

$$\frac{\partial Y}{\partial \tau} = -XZ + rX - Y + a,$$

$$\frac{\partial Z}{\partial \tau} = XY - bZ.$$

Sévellec & Fedorov, 2014
J. Climate
Mosto et al, 2025, *Chaos*
Le Bras et al, 2024,
Nonlinear processes in Geophysics

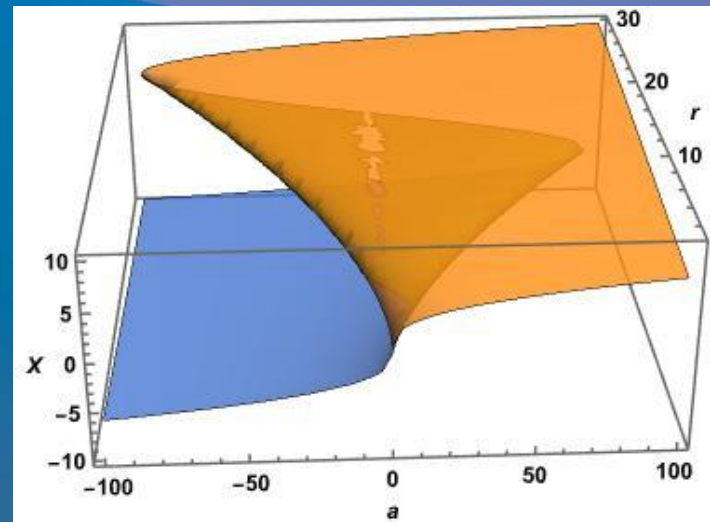
$$a = \pm \frac{2\sqrt{b}(r-1)^{\frac{3}{2}}}{3\sqrt{3}}$$

- Double fold bifurcation

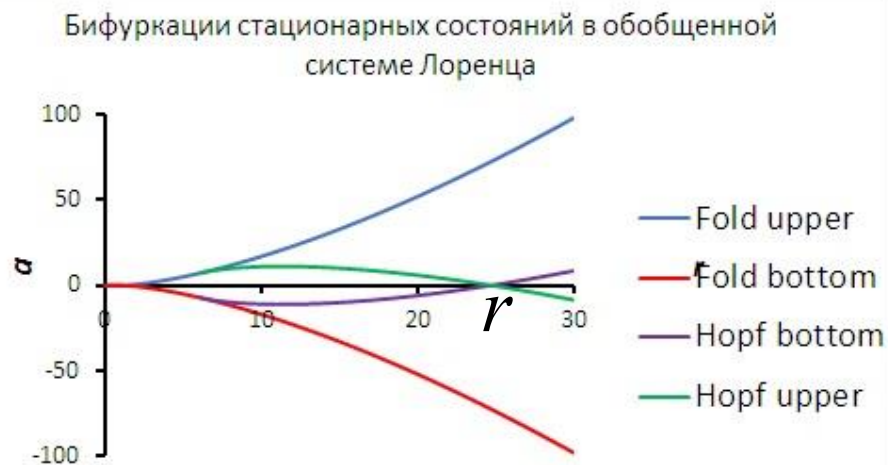
Bogdanov-Takens bifurcations

$$r_{cr} = \frac{2(b+\sigma)+3b\sigma}{2\sigma-b},$$

$$a_{cr} = \pm 2b^2 \left(\frac{\sigma+1}{2\sigma-b} \right)^{3/2},$$



Стационарные состояния X в зависимости от параметров a и r



Critical Slowdown (CSD) and Early Warning Signals



Признаки приближения к опасной границе

В окрестностях особых точек –
Critical Slowdown – Замедление движений =>
Увеличение дисперсии и рост автокорреляций

Formal description of fold CSD

Normal form of fold + stochastic forcing

$$dX = (\mu - X^2)dt + \sigma dW_t,$$

Tipping point: $\mu = 0$

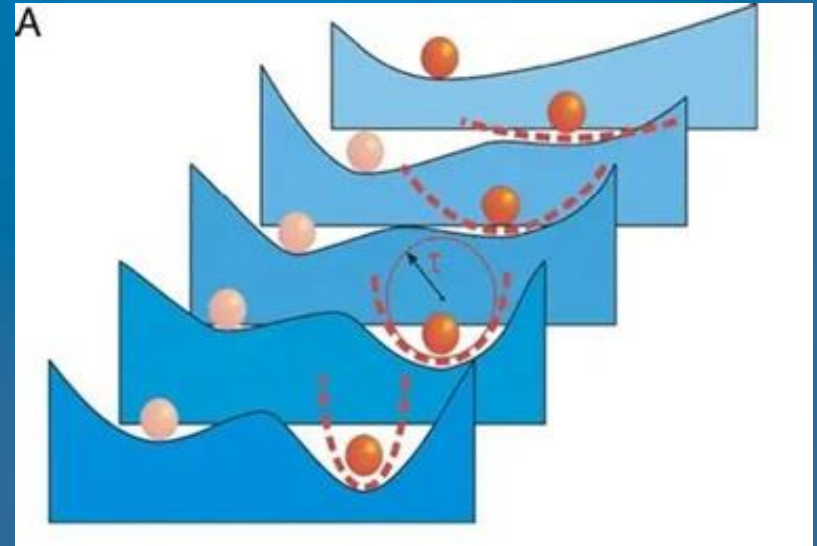
Into vicinity of stable point $X_1 = \sqrt{\mu}$

$$d(X - X_1) = -2\sqrt{\mu} (X - X_1)dt + \sigma dW_t,$$

– процесс Орнштейна-Уленбега, =>

$$Var[X] = \frac{\sigma^2}{4\sqrt{\mu}},$$

$$AC(X(s), X(t)) = \exp(-2\sqrt{\mu} |t - s|).$$



Lenton et al, 2008, *PNAS*

Peter Ditlevsen & Susanna Ditlevsen, 2023,
Nature Communications

Прогнозное время коллапса АМОС 2057г
с 95% доверит. границами 2025-2095.

Splitting of trajectories and lost of climatic predictability, 1

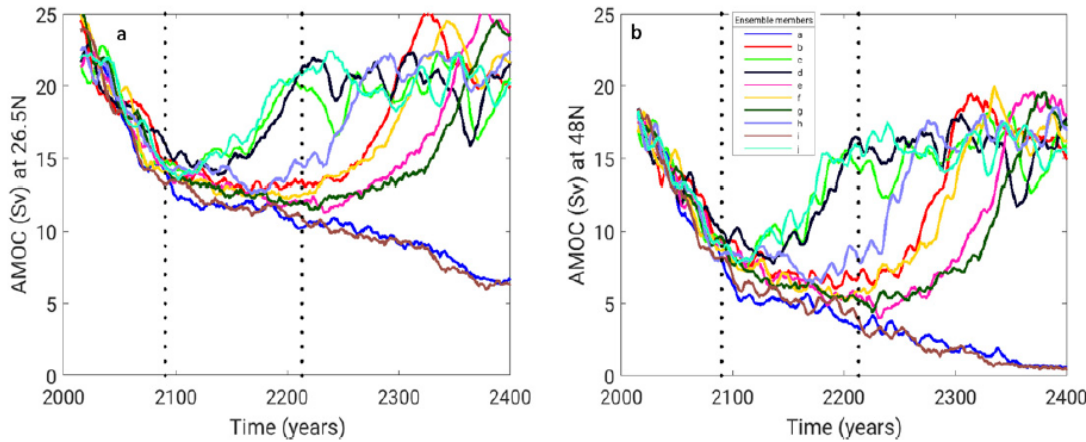
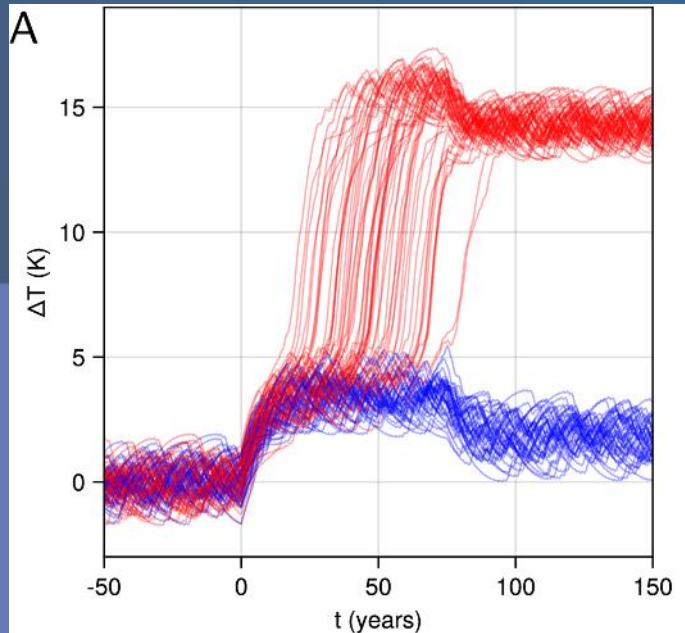


FIG. 1. AMOC strength, defined as the maximum overturning streamfunction below 500 m at (a) 26.5° and (b) 48°N, from 10 ensemble members of the SSP2-4.5 scenario simulation. Vertical dotted lines correspond to times of change in GHG forcing (see Fig. 2a). All fields are smoothed using a 10-yr moving average filter.

AMOC strength (Sv) at 26° and 48°N,
GISS-E2-1-G, SSP2-4.5, 10 ensemble members.
Romanou et al, 2023,
J.Climate



Predictability of possible storylines – Probabilistic prediction of future attractor

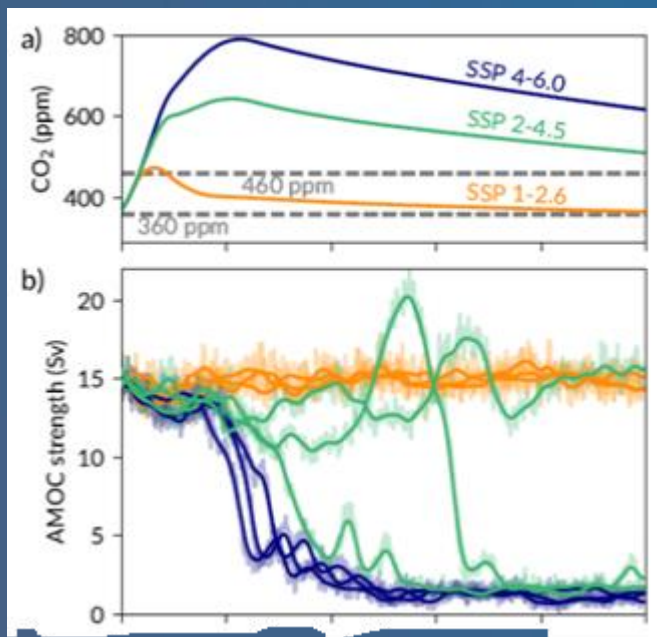
Ensemble of 100 simulations, uniformly placed initial conditions for $t = -100$ yr to a quadrupling of CO₂ in the period 0–75 yr and halving thereafter;

ΔT - the change from the mean temperature before the forcing. If $\Delta T > 5$ red, If $\Delta T < 5$ blue.

GEBM of Budyko–Sellers–Ghil type + Lorenz 63 as forcing.

Lohmann et al, 2024, *Journal of Physics: Complexity*

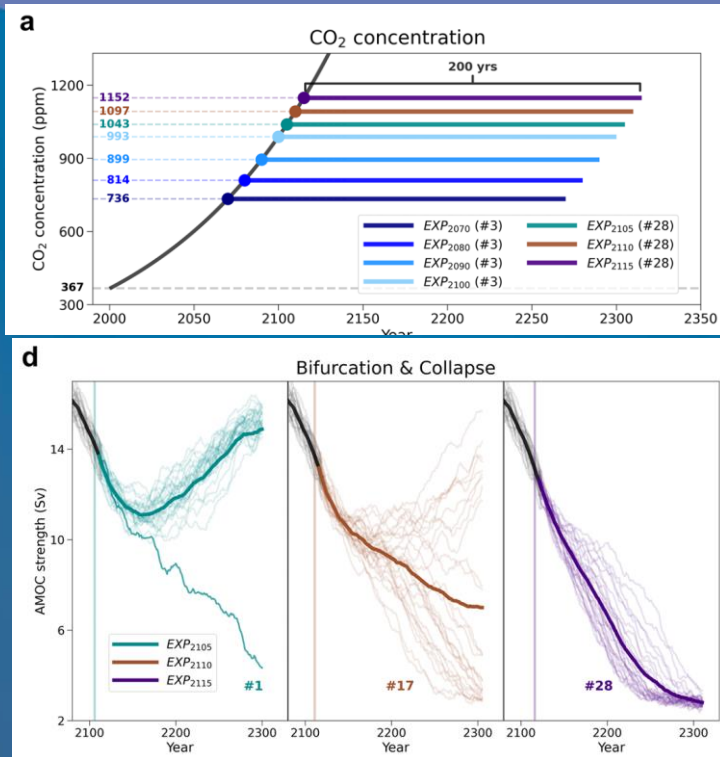
Splitting of trajectories and loss of climatic predictability



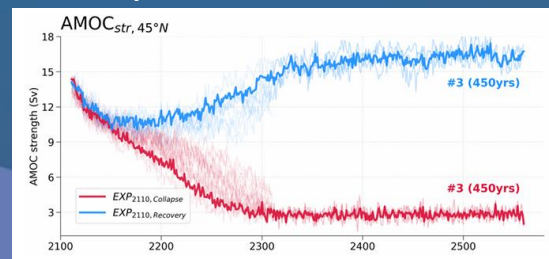
AMOC in PlaSim-LSG under three SSP scenarios from 2000 to 3000 CE.

- a) Atmospheric CO₂ concentration
- b) AMOC strength for simulations (three ensemble members each).

Börner, Mehling, Hardenberg & Lucarini, 2025



Design of CO₂ stabilization simulations and their AMOC response. CESM-1



Oh et al, 2025, Nature Communications

Фрактальность бассейнов притяжения в системе Lorenz-63

$$r = 20, \quad b = 8/3, \quad \sigma = 10$$

Стационарные состояния

$$Z_c = r - 1, \quad X_c = Y_c = \pm\sqrt{b(r-1)} \approx \pm 7.12$$

Притяжение к "+" голубой цвет, к "-" белый

Телескопизация –

Вверху X,Y диапазон (-40, 40)

середн X,Y (-4, 4)

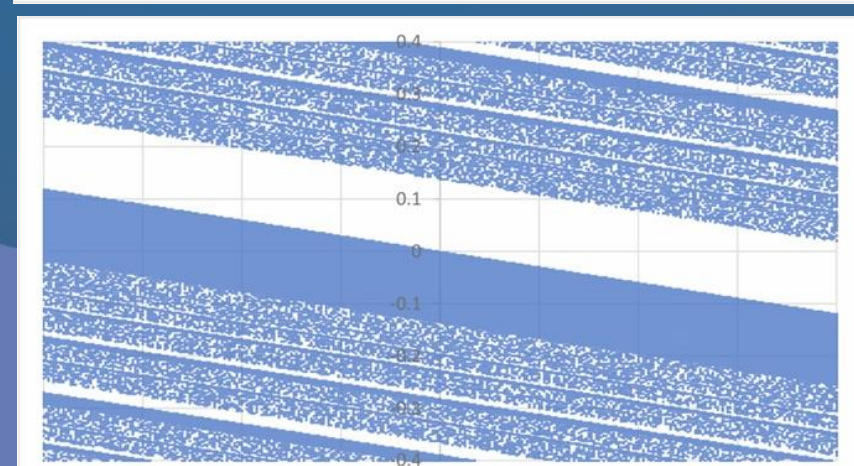
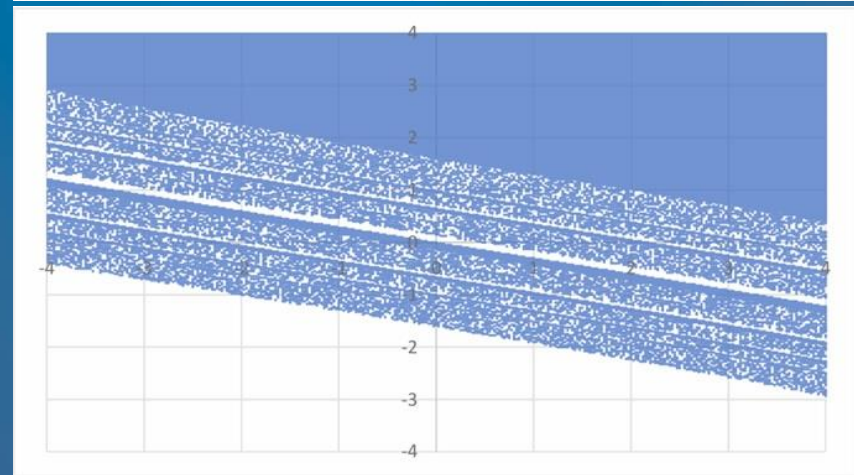
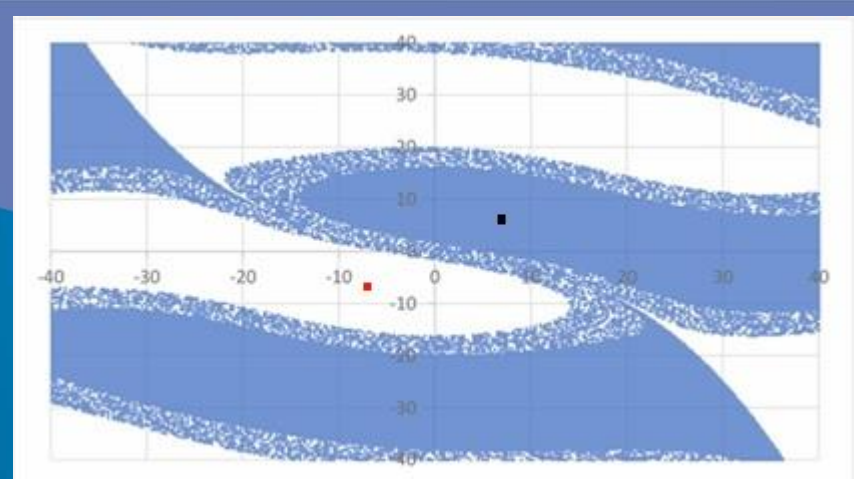
внизу X,Y (-0.4, 0.4)

Фрактальные границы бассейнов аттракторов →

Потеря предсказуемости второго рода

Kaszás, Feudel & Tél, 2019, *Scientific reports*

...fractality-induced tipping... as a consequence of the ubiquitous presence of fractal basin boundaries.





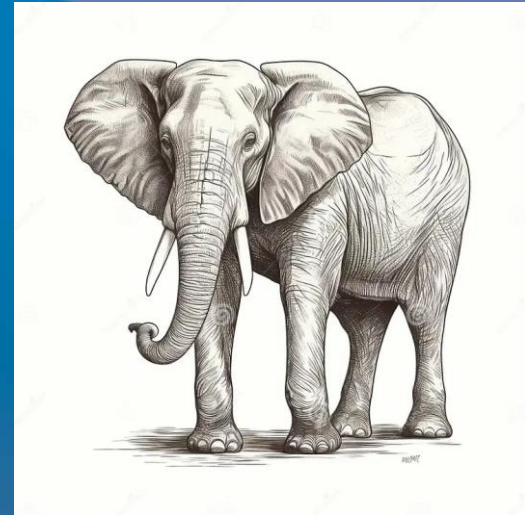
Climate Tipping Points – часть Nonlinear Climate Theory



Теория Климата



Теория Нелинейных
Динамических Систем



Статистическая
физика

Концепция ТР Красива, Интересна, Многогранна

Благодарю за внимание!

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